Nonlinear Control

Lecture # 38

Tracking & Regulation

Output Feedback

Tracking:

$$\begin{aligned}\dot{\eta} &= f_0(\eta, \xi)\\ \dot{e}_i &= e_{i+1}, \qquad 1 \le i \le \rho - 1\\ \dot{e}_\rho &= a(\eta, \xi) + b(\eta, \xi)u + \delta(t, \eta, \xi, u) - r^{(\rho)}(t)\end{aligned}$$

Regulation:

$$\begin{aligned}\dot{\eta} &= f_0(\eta, \xi, w)\\ \dot{\xi}_i &= \xi_{i+1}, \qquad 1 \le i \le \rho - 1\\ \dot{\xi}_\rho &= a(\eta, \xi, w) + b(\eta, \xi, w)u\\ y &= \xi_1\end{aligned}$$

- Design partial state feedback control that uses ξ
- Use a high-gain observer

Tracking sliding mode controller:

$$u = -\beta(\xi)\ \mathrm{sat}\left(\frac{k_1 e_1 + \cdots + k_{\rho-1} e_{\rho-1} + e_\rho}{\mu}\right)$$

Regulation sliding mode controller:

$$u = -\beta(\xi)\ \mathrm{sat}\left(\frac{k_0 e_0 + k_1 e_1 + \cdots + k_{\rho-1} e_{\rho-1} + e_\rho}{\mu}\right)$$

$$\dot{e}_0 = e_1 = y - r$$

β is allowed to depend only on ξ rather than the full state vector. On compact sets, the η-dependent part of $\varrho(\eta, \xi)$ can be bounded by a constant

High-gain observer:

$$
\begin{aligned}
\dot{\hat{e}}_i &= \hat{e}_{i+1} + \frac{\alpha_i}{\varepsilon^i}(y - r - \hat{e}_1), \quad 1 \le i \le \rho - 1 \\
\dot{\hat{e}}_\rho &= \frac{\alpha_\rho}{\varepsilon^\rho}(y - r - \hat{e}_1)
\end{aligned}
$$

$$\lambda^\rho + \alpha_1 \lambda^{\rho-1} + \cdots + \alpha_{\rho-1}\lambda + \alpha_\rho \text{ Hurwitz}$$

$$e \ \to \hat{e}$$

$$\xi \ \to \hat{\xi} = \hat{e} + \mathcal{R}$$

$$\beta(\hat{\xi}) \ \to \ \beta_s(\hat{\xi}) \text{ (saturated)}$$

Tracking:

$$u = -\beta_s(\hat{\xi}) \text{ sat}\left(\frac{k_1\hat{e}_1 + \cdots + k_{\rho-1}\hat{e}_{\rho-1} + \hat{e}_\rho}{\mu}\right)$$

Regulation:

$$u = -\beta_s(\hat{\xi}) \text{ sat}\left(\frac{k_0 e_0 + k_1\hat{e}_1 + \cdots + k_{\rho-1}\hat{e}_{\rho-1} + \hat{e}_\rho}{\mu}\right)$$

We can replace $\hat{e}_1$ by e_1

Special case: When β_s is constant or function of $\hat{e}$ rather than $\hat{\xi}$, we do not need the derivatives of r, as required by Assumption 13.4

The output feedback controllers recover the performance of the partial state feedback controllers for sufficiently small ε. In the regulation case, the regulation error converges to zero

Relative degree one systems: No observer

$$u = -\beta(y)\,\mathrm{sat}\left(\frac{y-r}{\mu}\right), \qquad u = -\beta(y)\,\mathrm{sat}\left(\frac{k_0 e_0 + y - r}{\mu}\right)$$

Example 13.5 (Revisit Example 13.2 and 13.4)

Use the high-gain observer

$$\dot{\hat{e}}_1 = \hat{e}_2 + \frac{2}{\varepsilon}(e_1 - \hat{e}_1), \quad \dot{\hat{e}}_2 = \frac{1}{\varepsilon^2}(e_1 - \hat{e}_1)$$

to implement the tracking controller

$$u = -(2|e_2| + 3)\ \mathrm{sat}\left(\frac{e_1 + e_2}{\mu}\right) \quad \text{(Example 13.2)}$$

and the regulating controller

$$u = -(2|e_1|+4|e_2|+4)\ \mathrm{sat}\left(\frac{e_0 + 2e_1 + e_2}{\mu}\right) \quad \text{(Example 13.4)}$$

Replace e_2 by $\hat{e}_2$ but keep e_1

Saturate $|\hat{e}_2|$ in the β function over a compact set of interest. There is no need to saturate $\hat{e}_2$ inside the saturation

For Example 13.2,

$$\Omega = \{|e_1| \le c/\theta\} \times \{|s| \le c\}, \quad c > 0,\ 0 < \theta < 1$$

is positively invariant. Take $c = 2$ and $1/\theta = 1.1$

$$\Omega = \{|e_1| \le 2.2\} \times \{|s| \le 2\}$$

Over Ω, $|e_2| \le |e_1| + |s| \le 4.2$. Saturate $|\hat{e}_2|$ at 4.5

$$u = -\left(2 \times 4.5 \ \mathrm{sat}\left(\frac{|\hat{e}_2|}{4.5}\right) + 3\right) \ \mathrm{sat}\left(\frac{e_1 + \hat{e}_2}{\mu}\right)$$

For Example 13.4,

$$\dot{\zeta} = A\zeta + Bs, \quad \zeta = \begin{bmatrix} e_0 \\ e_1 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$PA + A^T P = -I, 0 < \theta < 1, \rho_1 = \lambda_{\max}(P)(2\|PB\|/\theta)^2, c > 0$$

$$\Omega = \{\zeta^T P \zeta \leq \rho_1 c^2\} \times \{|s| \leq c\}$$

is positively invariant. Take $c = 4$ and $1/\theta = 1.003$

$$\Omega = \{\zeta^T P \zeta \leq 55\} \times \{|s| \leq 4\}$$

Over Ω,

$$|e_0 + 2e_1| \leq 22.25 \;\Rightarrow\; |e_2| \leq |e_0 + 2e_1| + |s| \leq 26.25$$

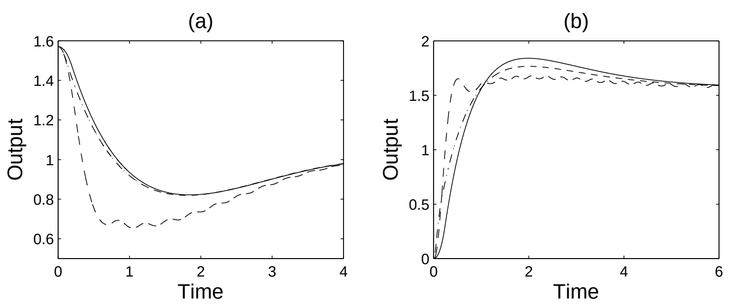
Saturate $|\hat{e}_2|$ at 27

$$u = -\left(2|e_1| + 4 \times 27 \,\mathrm{sat}\left(\frac{|\hat{e}_2|}{27}\right) + 4\right)\ \mathrm{sat}\left(\frac{e_0 + 2e_1 + \hat{e}_2}{\mu}\right)$$

Simulation; (a) Tracking, (b) regulation

$$\varepsilon = \begin{cases} 0.05 & \text{(dashed)} \\ 0.01 & \text{(dash-dot)} \end{cases}$$

State feedback (solid)


(a)
1.6
1.4
1.2
1
0.8
0.6
Output
0
1
2
3
4
Time
(b)
2
1.5
1
0.5
0
Output
0
2
4
6
Time